Spatial variability of stirring and mixing in the ocean

John Marshall, Ross Tulloch, Chris Hill, Shafer Smith

- coefficients of eddy diffusivity in the ocean exhibit great spatial variability
- very different K values are used to tune different models
- how does K vary spatially in the ocean?

Linear theory of mixing (Green 1970)

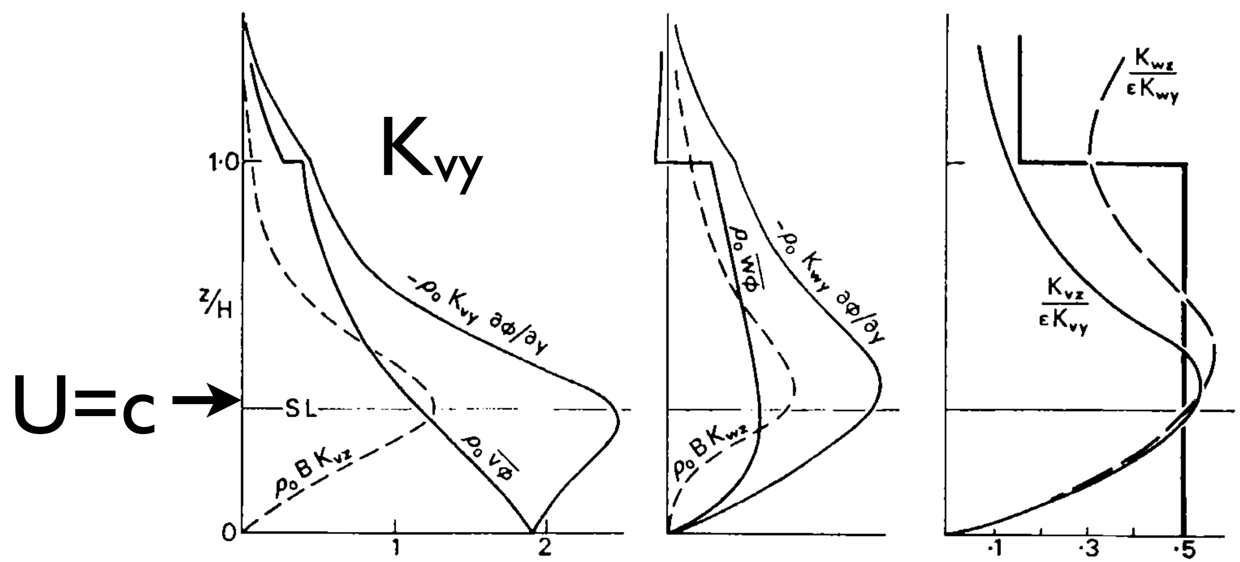
$$v = \text{Real}\{\exp i\lambda(x - ct)\,F(y, z)\}$$

hence $$\frac{d}{dt}\Delta y = \text{Real}\,\{\exp i\lambda\,(x_0 + (U - c)\,t)\,F(y, z)\} \quad + O(v^2)$$

and $$\Delta y = \text{Real}\left\{\frac{F}{i\lambda\,(U - c)}\exp i\lambda\,(x_0 + (U - c)\,t)\right\} + O(v^2)$$

$$K_{vy} = \overline{v\,\Delta y}^{x} = \frac{c_1\,|v|^2}{2\lambda\,|U - c|^2} + O(v^3) \quad \text{where} \quad c = c_0 + ic_1$$

Notice that K_{vy} as given by Eq. (15) is proportional to c_1 (so to the amplification rate) and is positive everywhere. Typically the variation of $|U - c|^2$ dominates over that of $|v|^2$ in the vertical and gives a pronounced maximum to K_{vy} close to the steering level, as illustrated in Fig. 8 (a).



(a) horizontal transfer (b) vertical transfer (c) slope of particle paths

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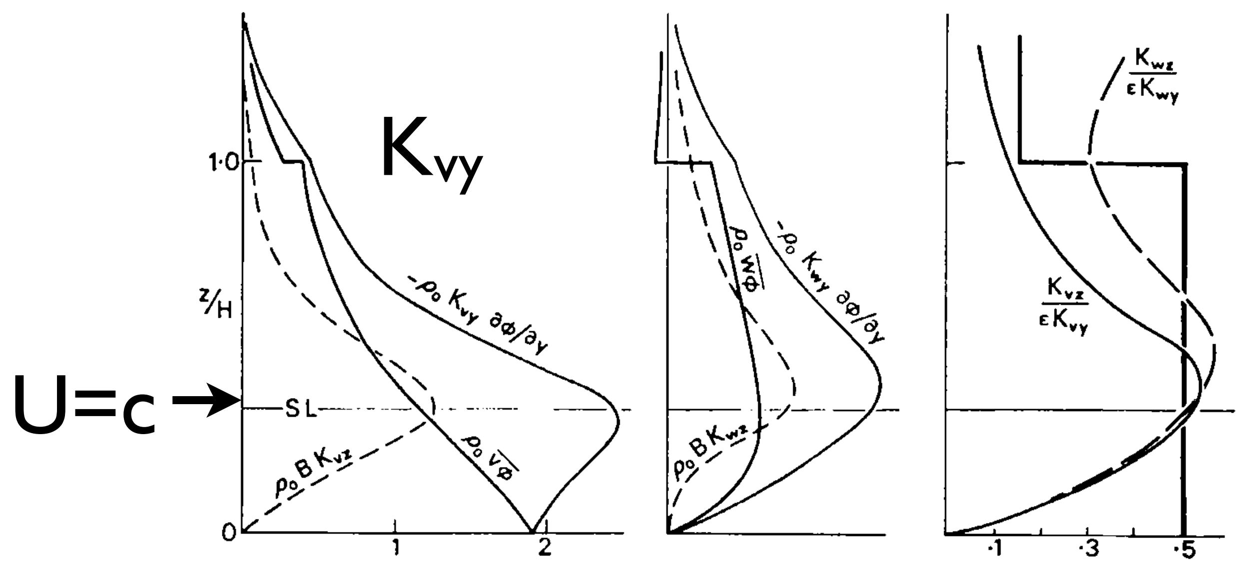
$$K_{vy} \sim \frac{EKE}{|U - c|^2}$$

"A useful guide"



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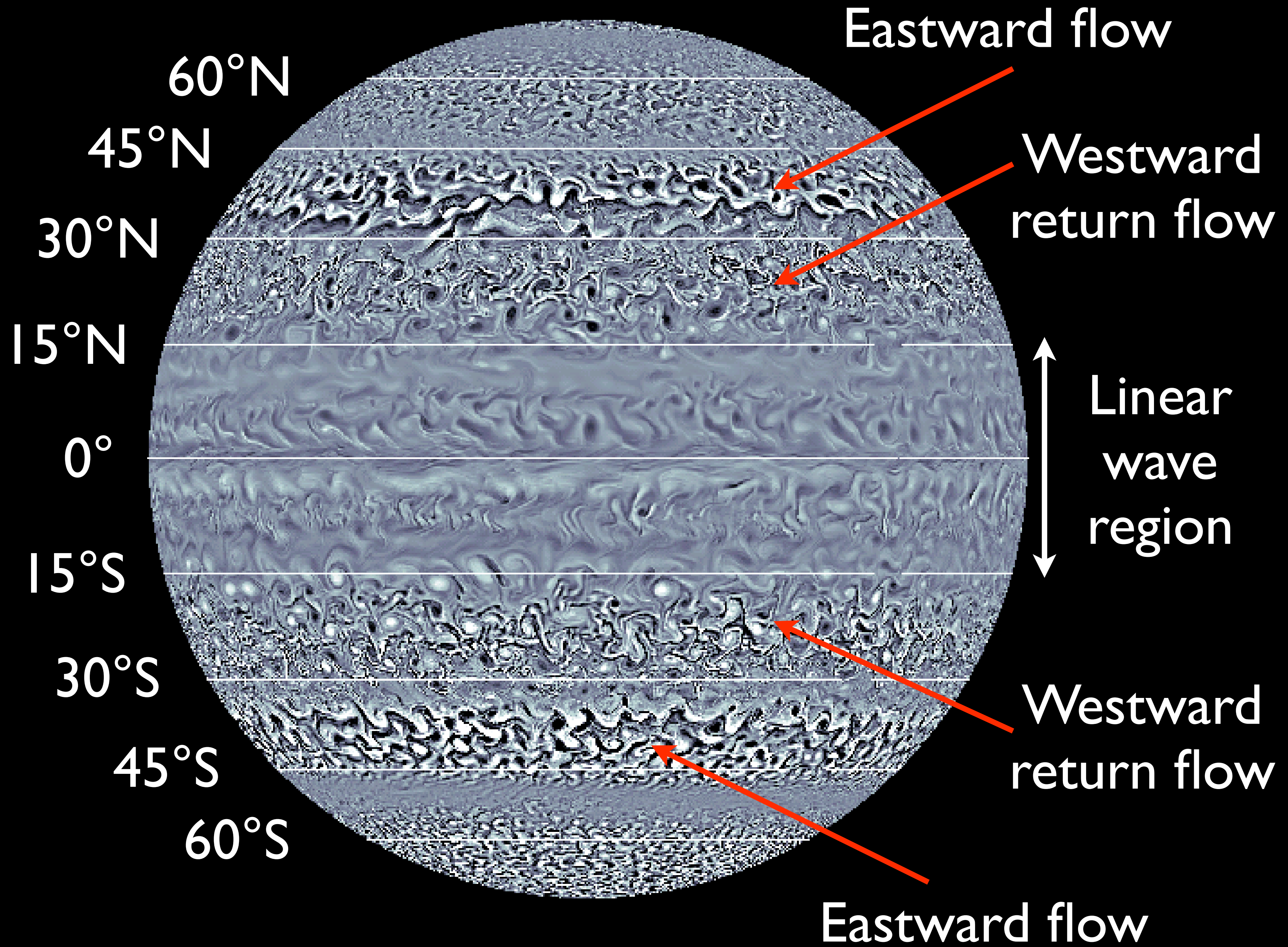
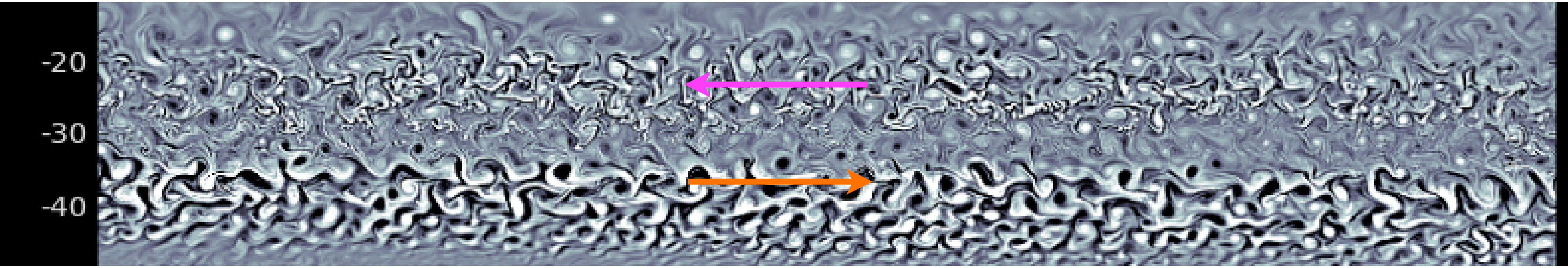
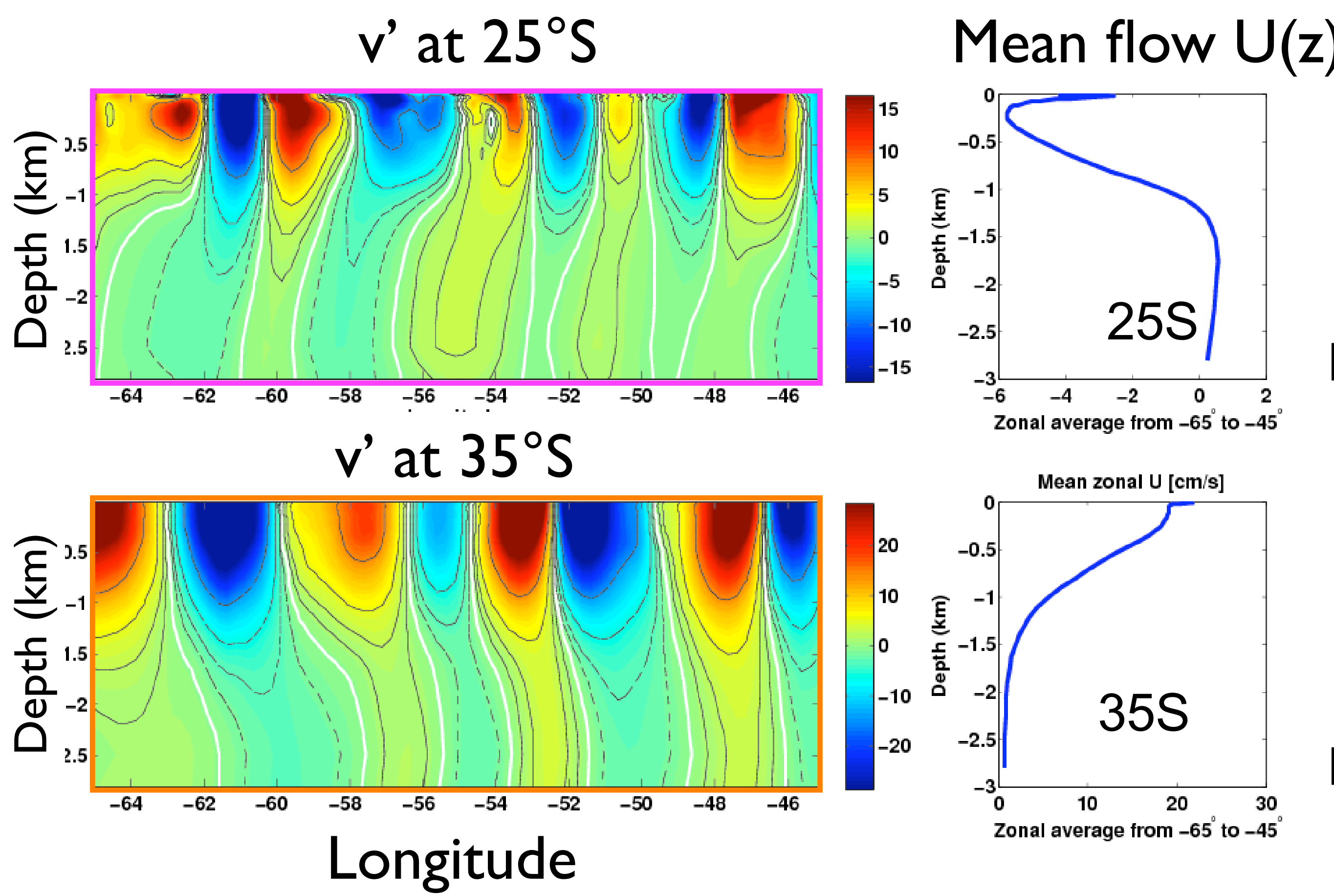
Lessons from an idealized eddy resolving model: The Double Drake







Easterly sheared mean flow

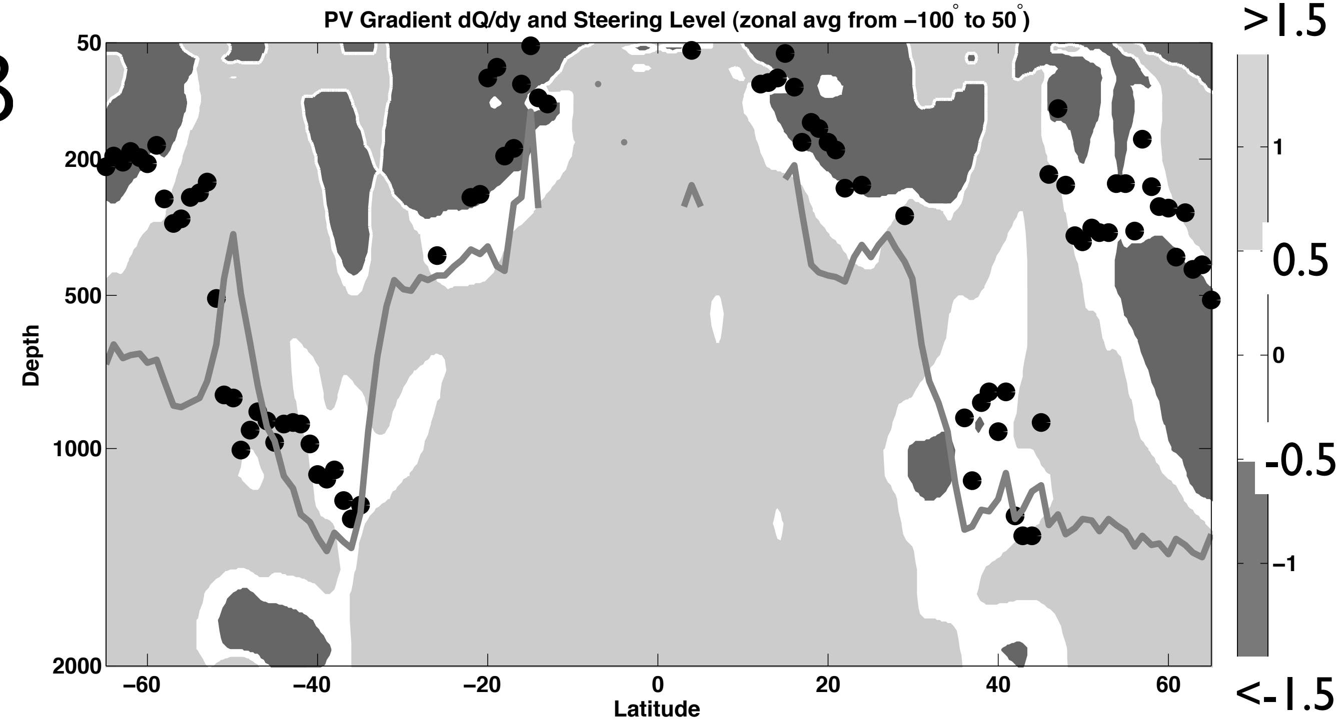
Westerly sheared mean flow

Eddying Double Drake

QGPV Gradient: Q_y/β

Black dots: depth at which $U_{mean}(z)=c_{phase\ model}$

Gray line: depth at which $u_{rms}(z)=c_{phase\ model}$



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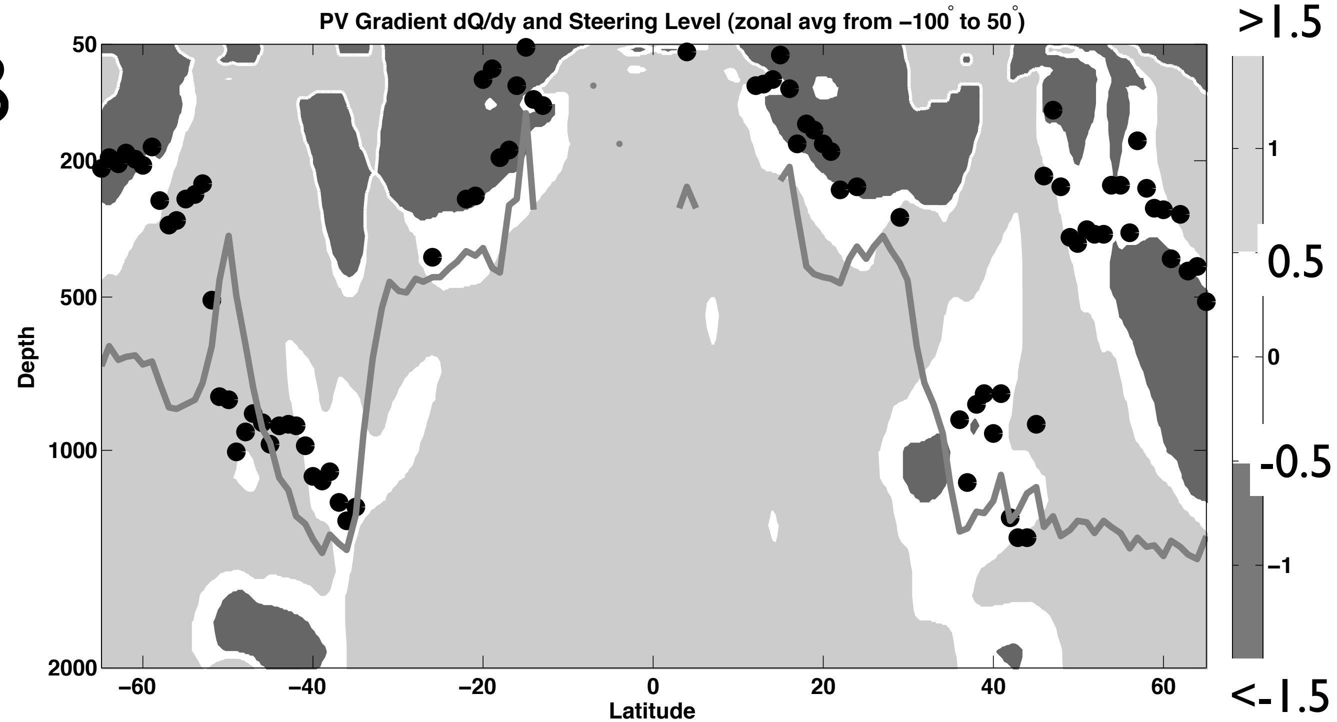
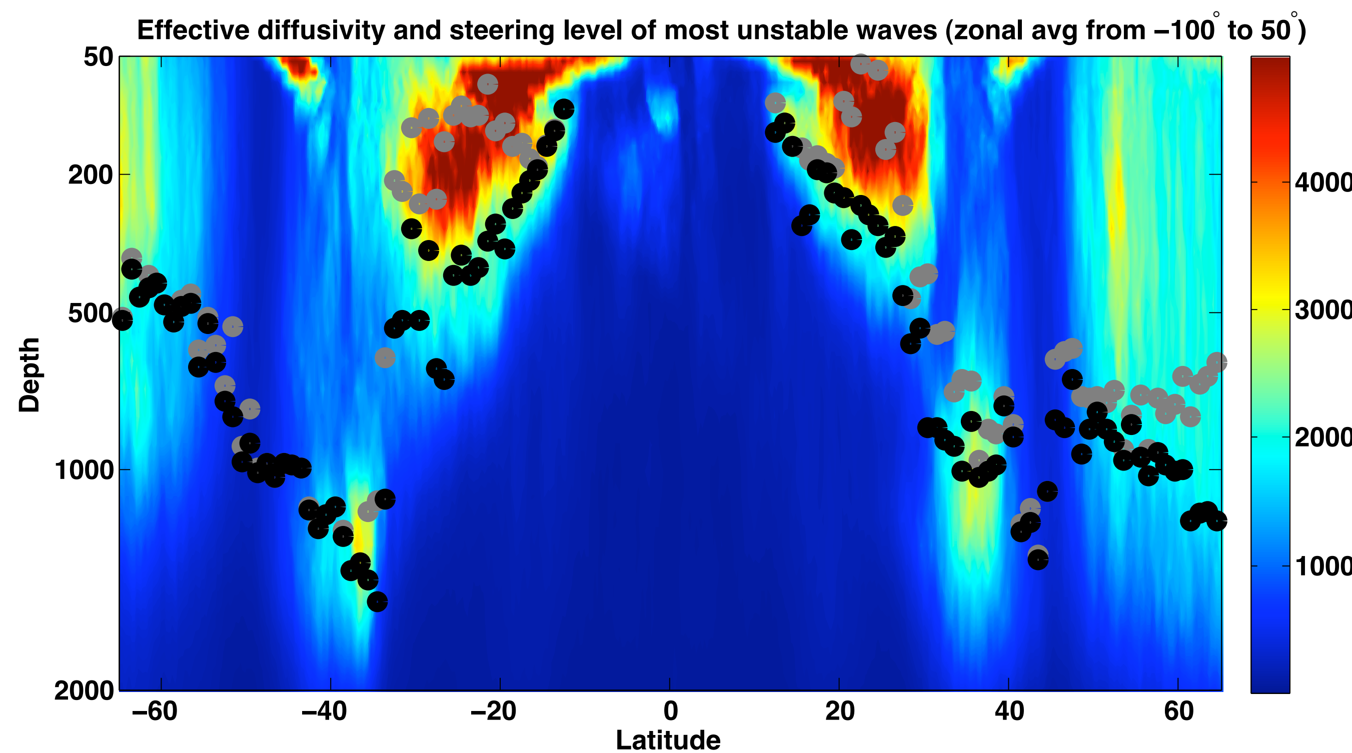


Effective diffusivity

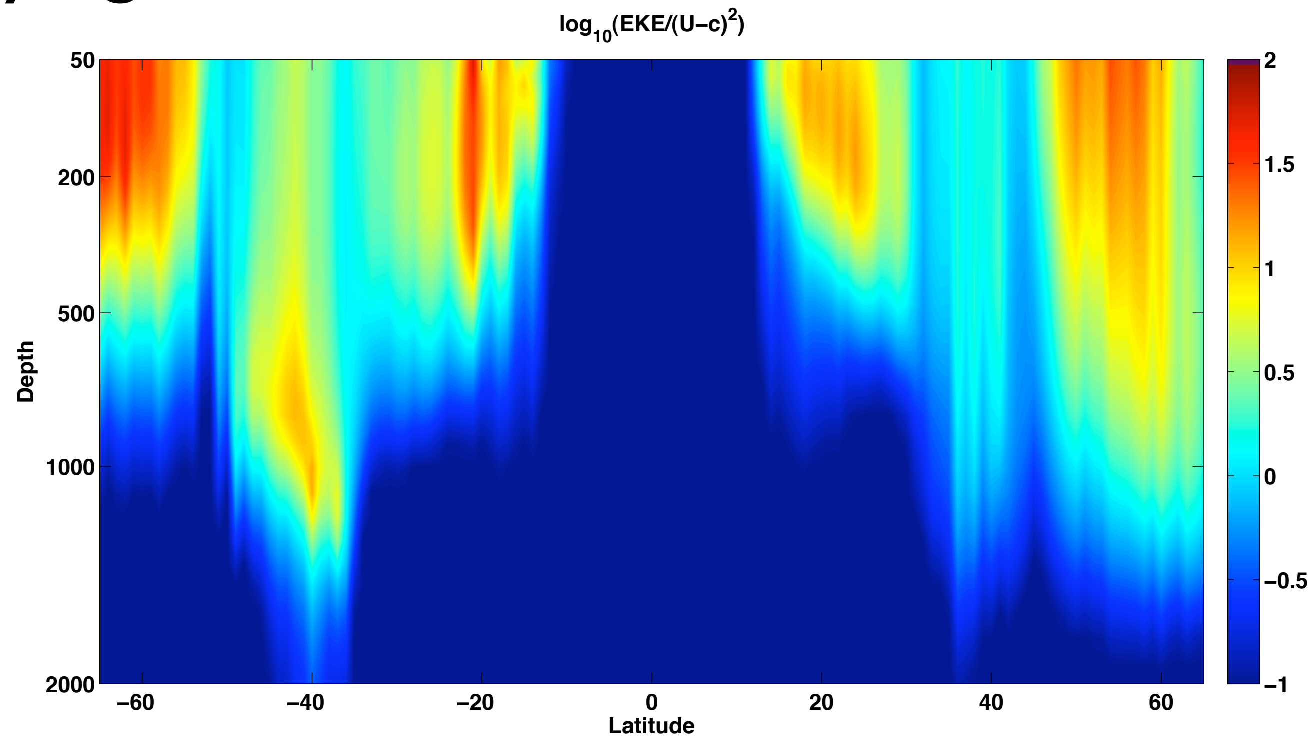
Dots: depth at which $U_{mean}(z) = c_{max(\omega i)}$



Eddying Double Drake

$EKE/(U_{mean}-c_{phase})^2$

(log scale)



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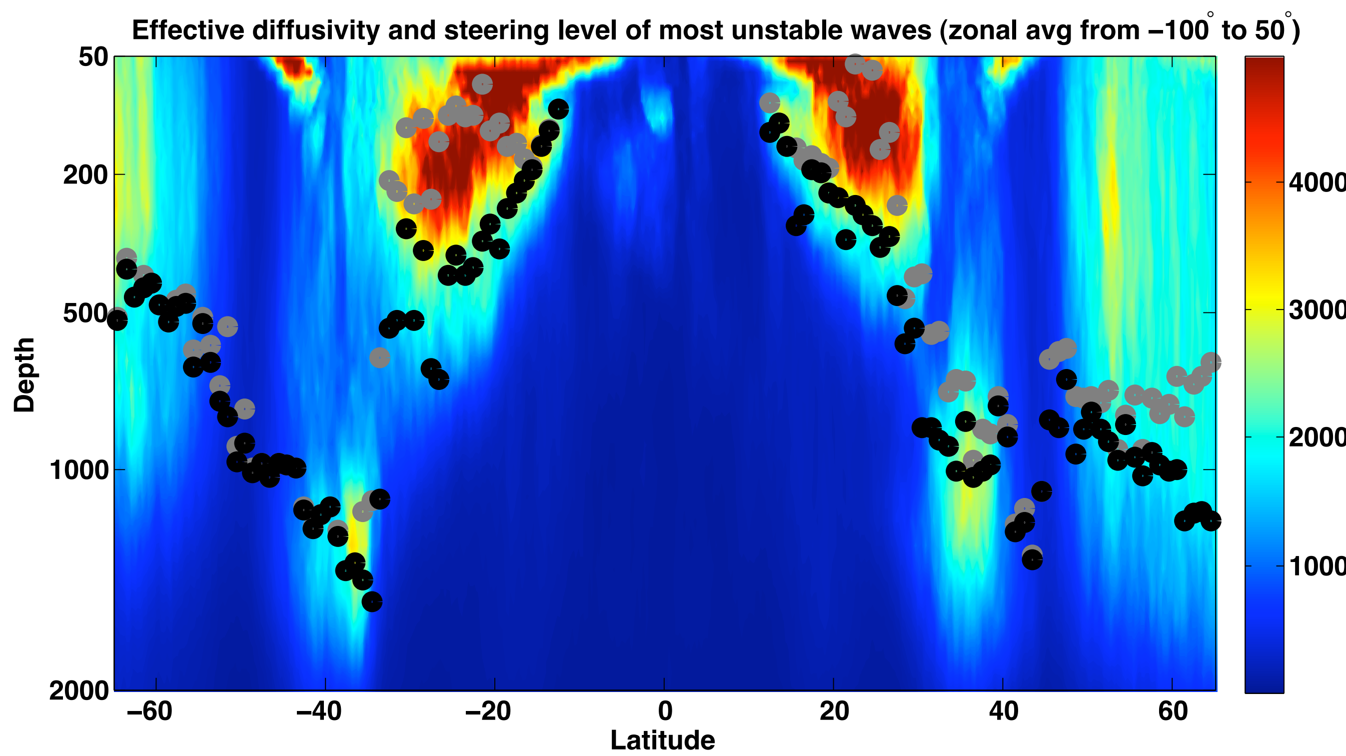
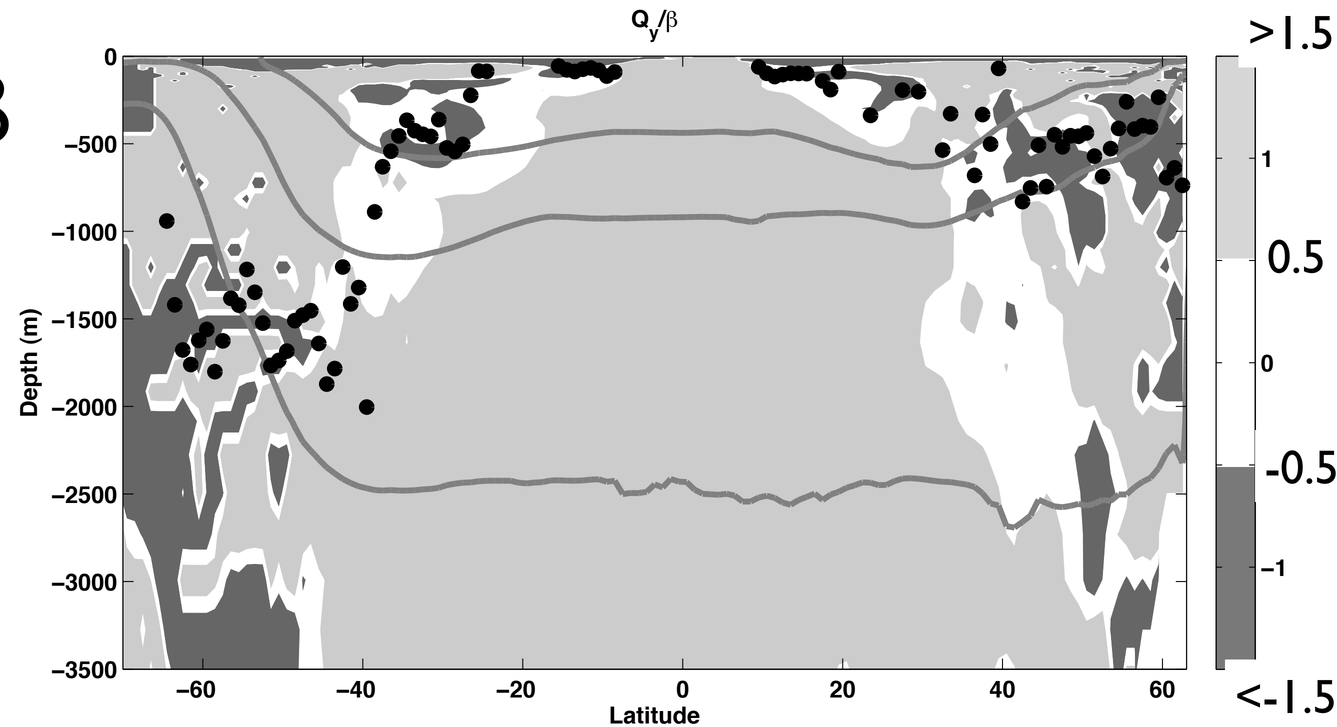


Ocean

QGPV Gradient: Q_y/β

Black dots: depth at which $U_{mean}(z)=c_{phase\ observed}$

Gray lines: isopycnals



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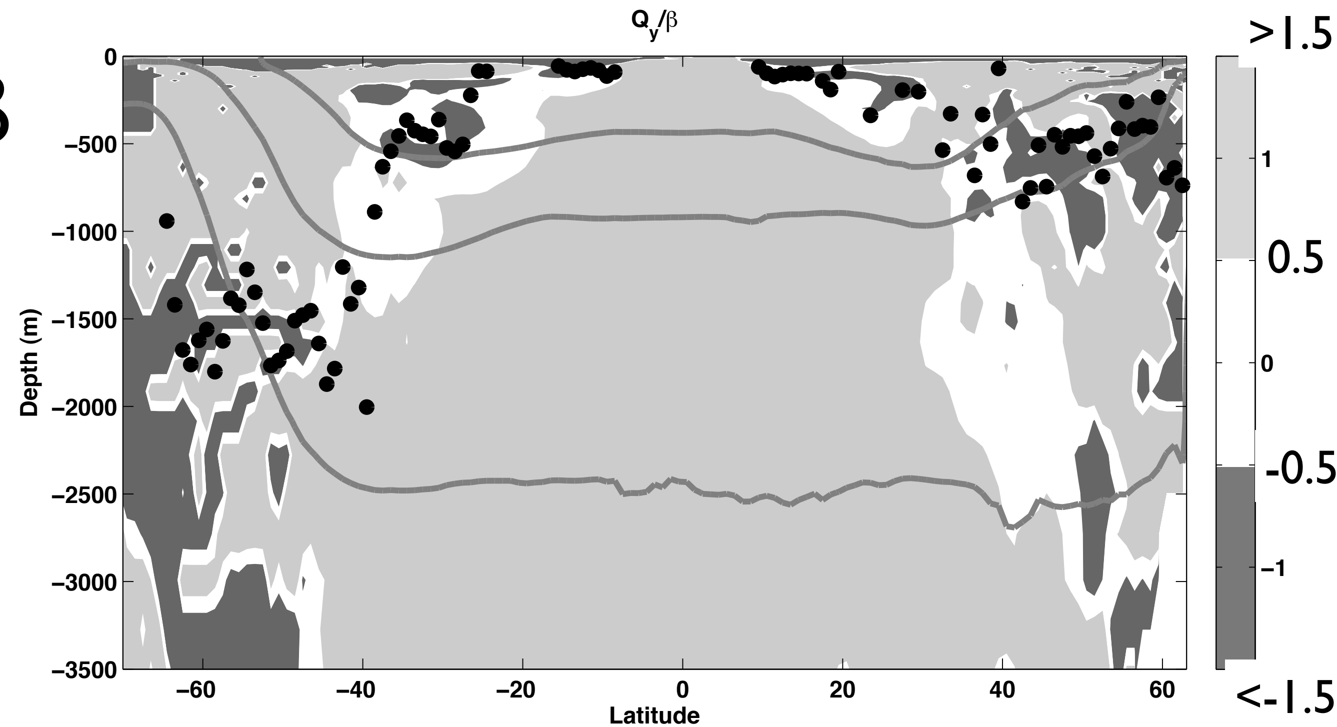
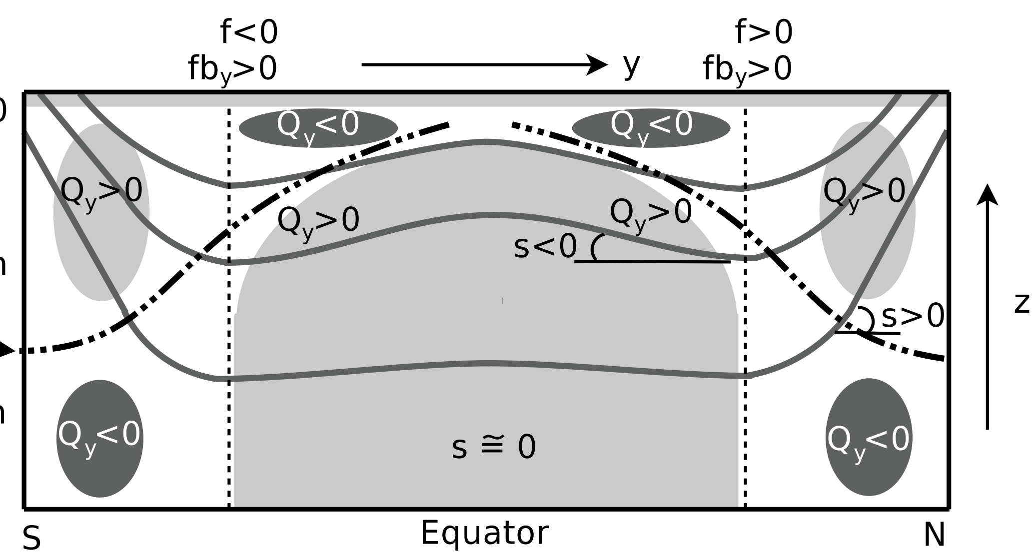


$$\frac{\partial \tilde{Q}}{\partial y} = \beta - f\frac{\partial s}{\partial z} + \frac{f^2}{N^2}\frac{dU}{dz}\delta_{\text{upper}}$$

$Q_y=0$



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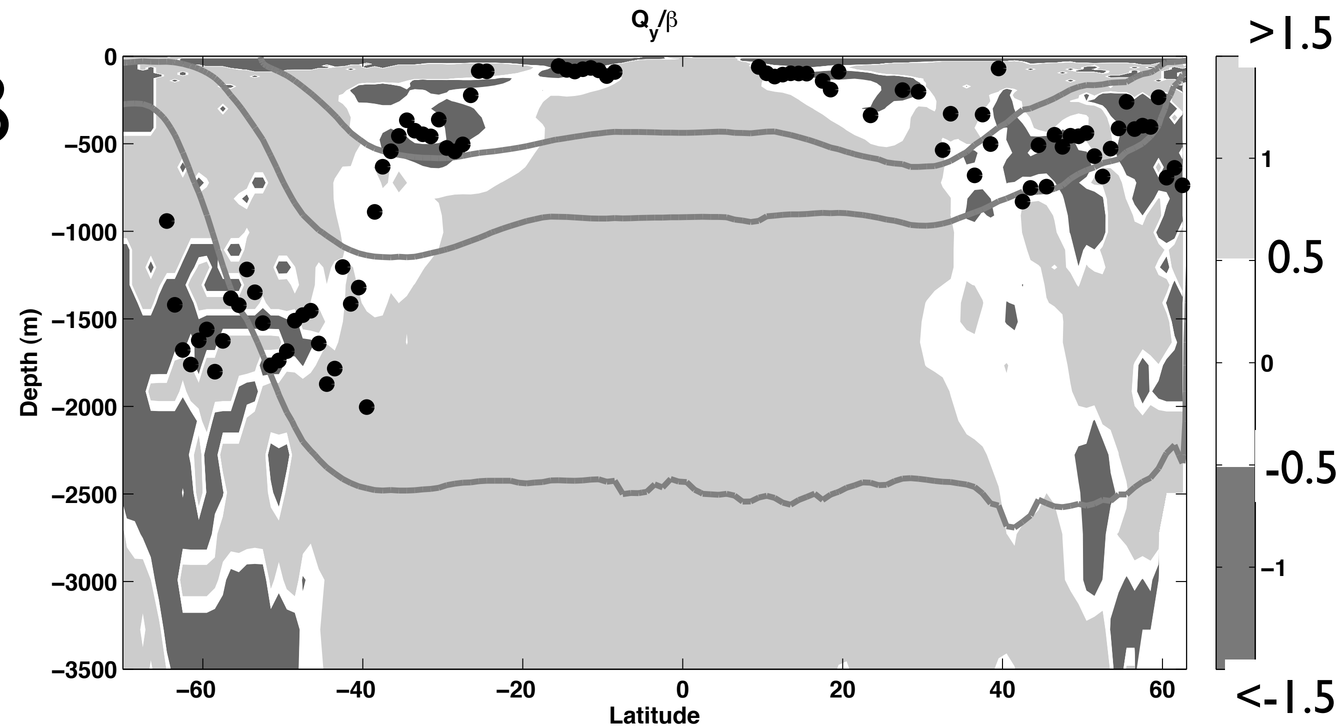
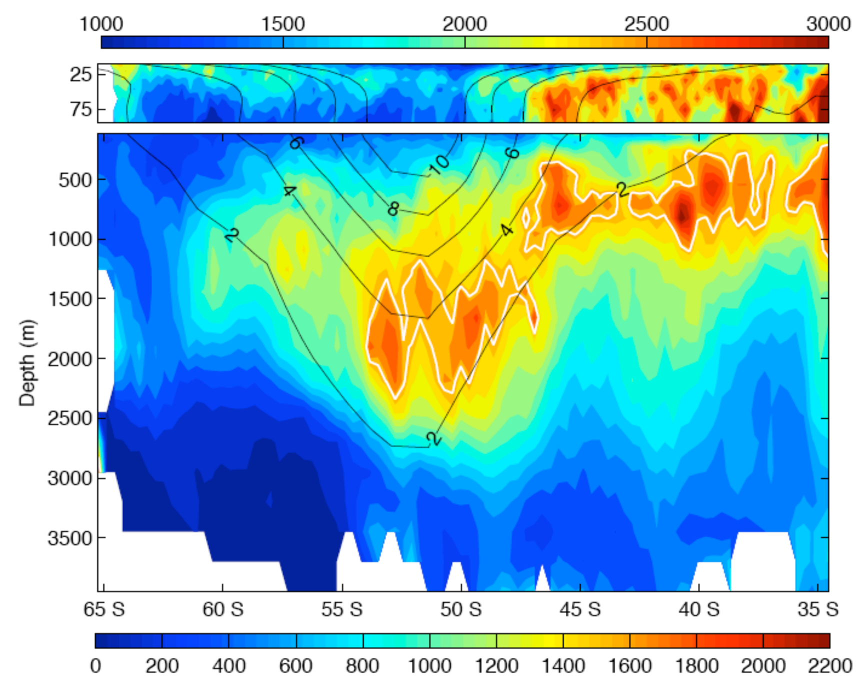


Effective Diffusivity in the ACC (isopycnal & streamwise averaged from the SOSE state estimate)

$\Rightarrow$ critical layer enhancement of mixing occurs in the ACC



Abernathey et al. (2009)