Sensitivities of sea-ice export through the Canadian Arctic Archipelago using MITgcm ocean/sea-ice adjoint model

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http://www.ecco-group.org
http://mitgcm.org
Long-term goal: truly global, high-resolution, coupled ocean/sea-ice state estimation

**ECCO2: High-Resolution Global-Ocean and Sea-Ice Data Synthesis @ NASA/Ames**

**MIT**
Marshall, Campin, Heimbach, Hill, Mazloff, Wunsch

**JPL**
Fu, Kwok, Lee, Menemenlis, Zlotnicki

**GSFC**
Rienecker, Suarez

**ARC**
Henze, Taft

**HARVARD**
Tziperman, Zanna

**GFDL**
Adcroft

**ARGONNE**
Hovland, Utke

ECCO: 1992 - 2002
c6ncp10

Jan 1992
ECCO2 Arctic Ocean studies

Arctic cost function reduction

2007-2008 summer sea ice minima

Nguyen et al., 2009 & Nguyen et al., in preparation
The MITgcm sea-ice model

• Thermodynamics
  – Based on Zhang & Hibler, 1997
  – Two-category, zero-layer, snow melting and flooding
    (Semtner, 1976; Washington & Parkinson, 1979)
  – Sea ice loading and dynamic ocean topography
    (Campin et al., Ocean Modelling, 2008)

• Dynamics
  – Both ported on C-grid for use in generalized curvilinear grids
  – Two solvers available for viscous-plastic (VP) rheology:
    • Line Successive Relaxation (LSR) implicit (Zhang & Hibler, 1997)
    • Elastic Viscous-Plastic (EVP) explicit (Hunke & Dukowicz, 1997)
  – Various advection schemes available

• An exact (with respect to tangent linearity) adjoint,
  – generated via automatic differentiation tool TAF

• Losch et al. (submitted to Ocean Modelling, 2009a)
• Heimbach et al. (submitted to Ocean Modelling, 2009b)
Sea Ice - Ocean coupling with $z^*$ coordinate

"Salt flux"

Campin, Marshall and Ferreira
Ocean Modelling, 2008
The forward model - configuration sensitivities
Ice drift velocities

- **C-grid**
- **LSR**
- **no-slip (C-LSR-ns)**

- **B-LSR-ns** minus **C-LSR-ns**

- **C-LSR-fs** minus **C-LSR-ns**

- **C-EVP-ns** minus **C-LSR-ns**
The MITgcm/sim adjoint models generated via Automatic Differentiation (AD)

- Model code
  \[
  \bar{v} = M_\Lambda \left( M_{\Lambda-1} \left( \ldots \left( M_0 (\bar{u}) \right) \right) \right)
  \]

- Adjoint code
  \[
  \delta^* \bar{u} = M_0^T \cdot M_1^T \cdot \ldots \cdot M_\Lambda^T \cdot \delta^* \bar{v}
  \]

- Automatic differentiation:
  - each line of code is elementary operator \( M_\lambda \)
  - \( \longrightarrow \) rules for differentiating elementary operations
  - \( \longrightarrow \) yield elementary Jacobians \( M_\lambda \)
  - \( \longrightarrow \) composition of \( M_\lambda \)'s according to chain rule
  - yield full tangent linear / adjoint model

- TAMC / TAF source-to-source tool (Giering & Kaminski, 1998)

- model \( M \)
- independent \( \bar{u} \)
- dependent \( \mathcal{J} \)

\[
\begin{align*}
\text{TAMC / TAF} & \quad \text{TLM } M, \text{ or } \text{ADM } M^T, \text{ or }\\
\{ \text{model } M \} & \quad \{ \text{TLM } M, \text{ or } \text{ADM } M^T, \text{ or } \} \text{gradient } \delta^* \bar{u} = \bar{\nabla}_u \mathcal{J}
\end{align*}
\]
The coupled ocean/sea-ice adjoint

Sensitivity of ice export to all elements in the coupled state:

- **sea-ice** (e.g. thickness, concentration, snow cover)
- **ocean** (temperature, salinity, velocities)
- **atmospheric boundary condition** (SAT, specific humidity, precipitation, shortwave radiation, wind velocity)
Arctic configuration

- Coarsened Arctic face of the ECCO2 global cubed sphere (from ~18 km to ~36 km horizontal resolution)
- Underlying ocean model uses various parameterization schemes (KPP, GM/Redi)
- 6-hourly forcing via NCEP/NCAR atmospheric state, converted to open-ocean air-sea fluxes via Large & Yeager (2004)
- Sea-ice dynamics via LSR on C-grid
- Adjoint runs on 80 processors (e.g. IBM SP, SGI Altix)
The Canadian Arctic Archipelago
Adjoint sensitivity of solid (snow & ice) freshwater transport through Lancaster Sound

\[
\frac{dJ}{dh_c} \quad [m^2s^{-1}/m]
\]

with

\[ h: \text{ effective thickness} \]
\[ c: \text{ ice concentration} \]
Perturbed - unperturbed ice export, testing the adjoint

Longitude-time diagrams of sensitivities (slice through Lancaster Sound)

- ice thickness
- SST
- precip.
Origin of sign change in precipitation sensitivities

perturbation on 1991/11/01

perturbation on 1991/04/01

Δθ (°C)

ΔSST (°C)

Δshortwave (W/m²)

Δlens (m)

Δlens (m)

Δlens (m)

Δlens (m)

Δlens (m)

Δlens (m)

Δlens (m)
Some Results & Outlook

• Complement configuration sensitivities (e.g. free-slip vs. no-slip boundary conditions) through aspects related to state space

• Adjoint model generated via automatic differentiation

• Adjoint sensitivities reveal pathways of ice export influences as function of underlying ocean/atmosphere state

• May reveal unexpected sensitivity behavior (e.g. here, sign of precipitation sensitivities)

• A crucial step to ascertain useful gradients for state estimation, which is the ultimate goal

• Coupled problem ought to propagate sensitivities across the model components;
  ➔ could be explored in state estimation
  ➔ obs of one component constrain the other component
Outlook: Sea-ice state estimation in a limited-area setup of the Labrador Sea

- MITgcm with Curvilinear Grid
  - 30 km x 30 km → 30 km x 16 km
  - 23 vertical levels
- 1.5 layer dynamic-thermodynamic sea ice model with snow
  - Stress-Strain rate based on Hibler (1980) ellipse
- Open boundaries
  - Weak sponge layers at Southern and Eastern edges
- Resolved Labrador and Greenland Shelves
  - Critical for sea ice production and advection
  - Important for boundary currents
- Computational efficient
  - Parallel: 1 real hr/ simulated year on 6 nodes

Ocean State Estimation (data assimilation)

How to synthesize? Estimation/optimal control problem:
Use a model (MITgcm) and its adjoint:

DEPTH (m): 5
TIME: 01 JAN-2000 00

Assimilation (Adjoint) by ODAP

DATA SET: Tove

Temperature (Deg C)
Sensitivity calculations in forward or reverse

**Finite difference approach:**
- Take a “guessed” anomaly (SST) and determine its impact on model output (MOC)
- Perturb each input element (SST(i, j)) to determine its impact on output (MOC).

**Reverse/adjoint approach:**
- Calculates “full” sensitivity field
  \[
  \frac{\partial \text{MOC}}{\partial \text{SST}(x,y,t)}
  \]
- Approach:
  Let \( J = \text{MOC}, \ u = \text{SST}(i,j) \)
  \[
  \vec{\nabla}_u J(u) = \frac{\partial \text{MOC}}{\partial \text{SST}(x,y,t)}
  \]

Impact of **one input** on **all outputs**

Sensitivity of **one output** to **all inputs**