

# Western Boundary Sea-Level: A Theory, Rule of Thumb, and Application to Climate Models

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A NEW THEORY FOR WESTERN  
BOUNDARY SEA LEVEL      d  
Sea level rise is one of the most      u  
important and potentially dange      s

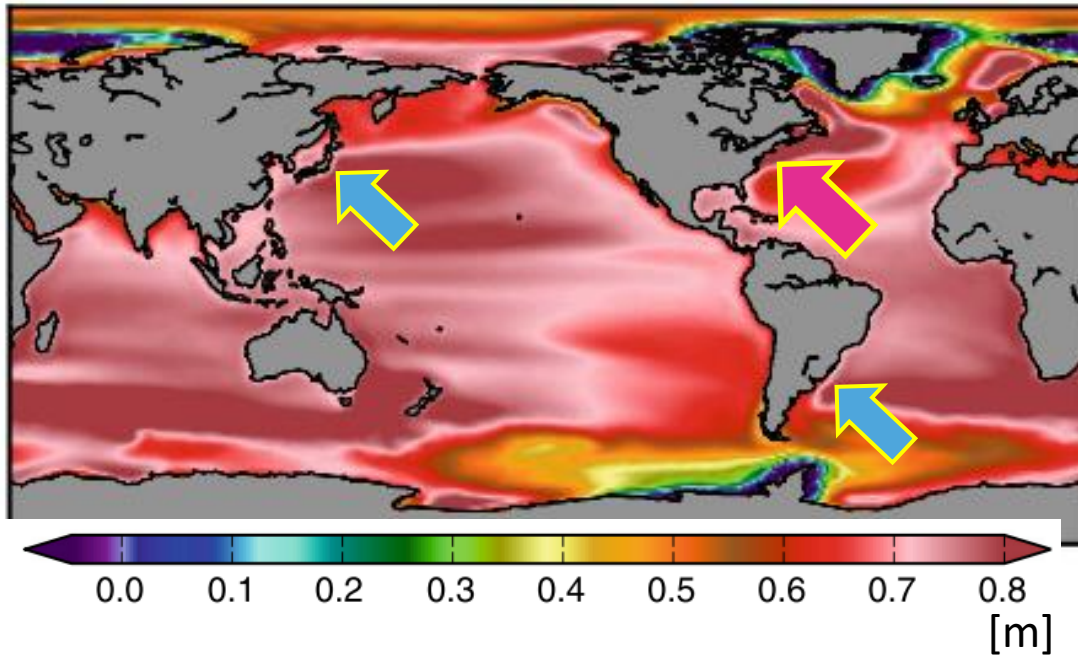
# Motivation

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# Western boundary sea-level rise

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Ocean (RCP8.5) + Land ice  
(RCP8.5) + Terrestrial water + GIA



Slangen et al. (2014)

High sea-level rise in the ocean interior near the western boundaries appear ...

- to reach the western coast of North Atlantic (sea-level rise hot-spot),
- Not to reach the western coast of South Atlantic & North Pacific.

At present, adequate explanations for the relationship between western boundary and ocean interior sea level are lacking.

# Previous studies for western boundary sea-level

- A group of studies treated western sea-level in order to close the **mass conservation over a basin** for QG (Liu et al., 1999) and a primitive reduced gravity model (Cessi and Louazel, 2001; Zhai et al. 2014).
- The other group of studies investigate western boundary sea-level more locally. In particular, Godfrey (1975, JPO) showed that **the western boundary sea-level can be expressed by the ocean interior sea-level** for a **straight meridionally running western boundary**.
  - The validity of his theory has not been examined.

# Purposes of this study are to

1. Expand Godfrey (1975)'s theory to slanted or curved meridional boundary
2. Validate the theory using a reduced gravity model
3. Explorer implications relevant to societal interest via Rule of Thumb
4. Apply the theory to CMIP5 model outputs.

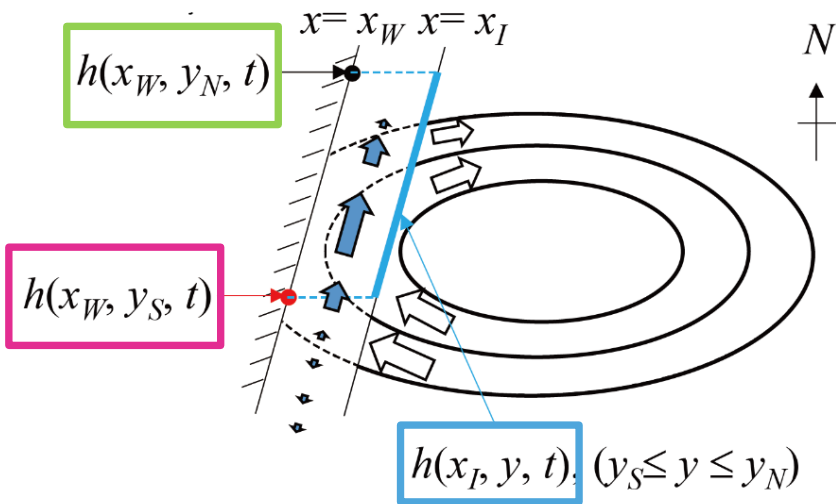
# Theory

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I skip the deviation of theory because of the time limitation.

# Final Eq. and Mechanism

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Mechanism:

Zonal mass input due to long Rossby wave is transmitted equatorward by the boundary layer transport.

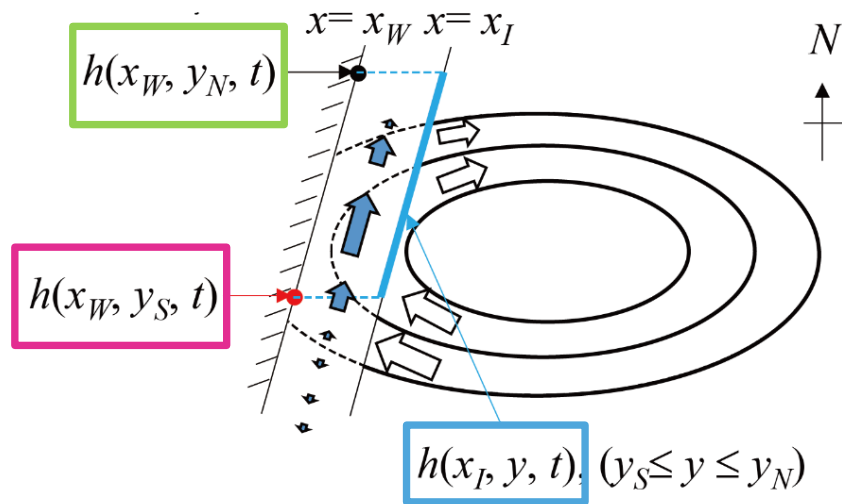
$$h(x_W(y_S), y_S, t) = \frac{f(y_S)}{f(y_N)} h(x_W(y_N), y_N, t) + f(y_S) \int_{y_S}^{y_N} \frac{\beta}{f^2} h(x_I(y), y, t) dy$$

Meaning:

The western boundary sea-level at  $y_S$  is contributed

1. from western boundary sea-level at  $y_N$
2. from interior sea-level between  $y_S$  and  $y_N$

# Final Eq. and Mechanism



## Mechanism:

Zonal mass input due to long Rossby wave is transmitted equatorward by the boundary layer transport.

$$h(x_W(y_S), y_S, t) = \frac{f(y_S)}{f(y_N)} h(x_W(y_N), y_N, t) + f(y_S) \int_{y_S}^{y_N} \frac{\beta}{f^2} h(x_I(y), y, t) dy$$

## No parameter with respect to oceanic vertical structure (e.g., $g'$ , $H$ )

- Reduced gravity model, barotropic one layer model, or any vertical mode (if vertical mode decomposition is applicable) satisfy this Eq.
- We can use this equation for understanding the mechanism of sea-level change map, without knowing vertical mode contributions.



# **Numerical Model and Comparison with the Theory**

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# Model and Experiment design

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- Model

- A linear, reduced gravity model with a B-grid on a sphere
- **h-point is at the land-sea boundary**, so that the coastal sea-level is a direct output.
- 0.5-deg grid spacing,  $70^\circ$  ( $10^\circ\text{S}$ - $60^\circ\text{N}$ ) width in meridional,  $100^\circ$  ( $0^\circ\text{E}$ - $100^\circ\text{E}$ ) width in zonal

- Steady-forcing experiments

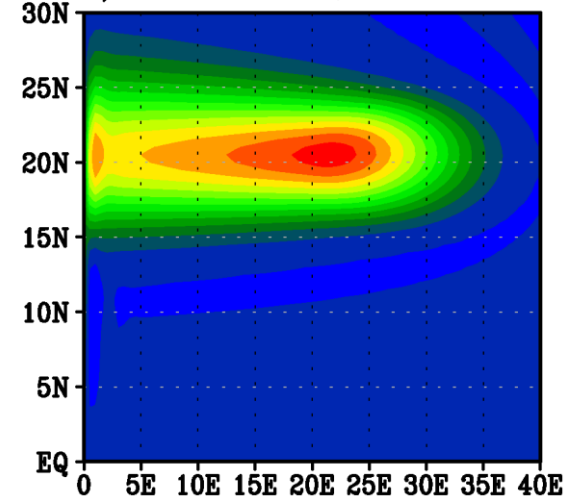
- Monopole mass input  $20^\circ$  width in zonal and  $10^\circ$  width in meridional centered at  $10^\circ\text{N}$ ,  $20^\circ\text{N}$ ,  $30^\circ\text{N}$ , or  $40^\circ\text{N}$
- Parameter change experiments with the forcing centered at  $20^\circ\text{N}$

- Initial-value experiment

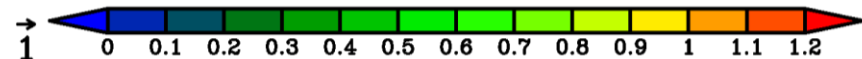
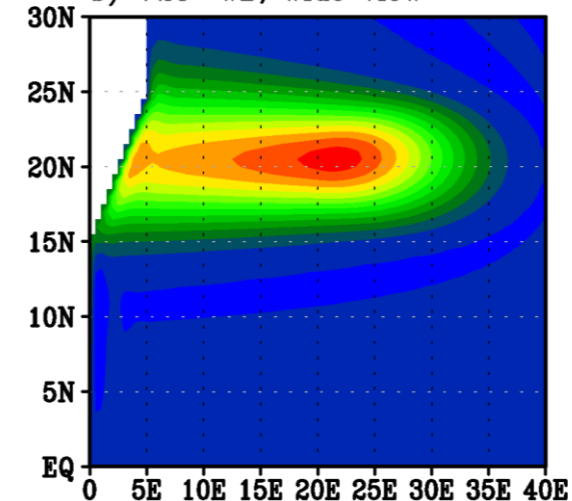
- Monopole initial value of h  $20^\circ$  width in zonal and  $10^\circ$  width in meridional centered at  $10^\circ\text{N}$ ,  $20^\circ\text{N}$ ,  $30^\circ\text{N}$ , or  $40^\circ\text{N}$ .

Steady forcing at centered  $20^\circ\text{N}$ ,  $30^\circ\text{E}$

a) Meridional WB, wide view



b)  $+25^\circ$  WB, wide view



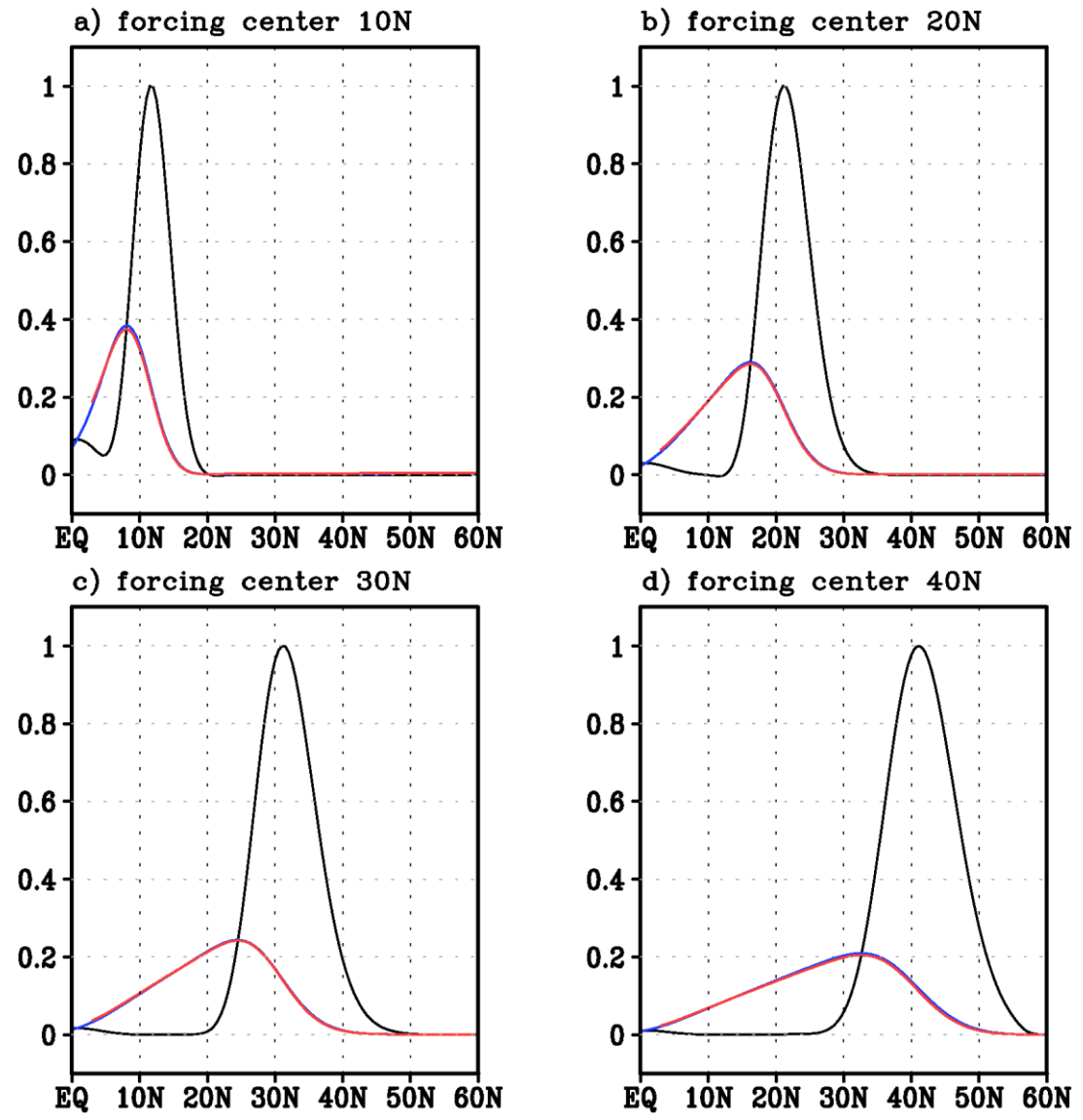
# Steady forcing, meridional western boundary

— 5°E (interior)  
 — 0°E (WB) model  
 — 0°E (WB) theory

Relative Root Mean Square Error [%]  
 between the theory and the numerical model

Latitude of Forcing Center			
10°N	20°N	30°N	40°N
2.9%	2.0%	1.7%	1.5%

Theory is very accurate.

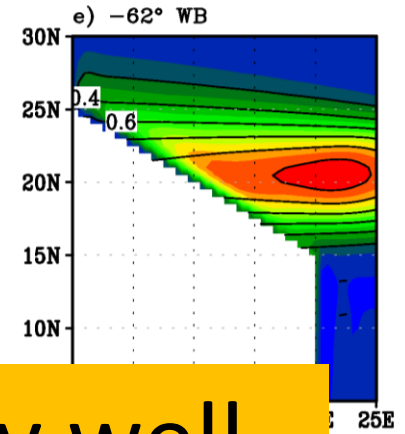
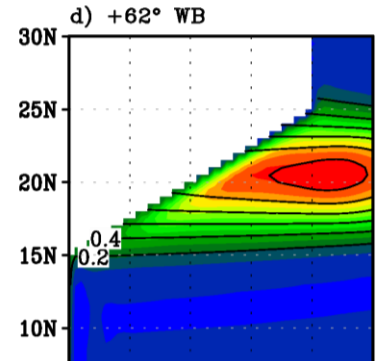
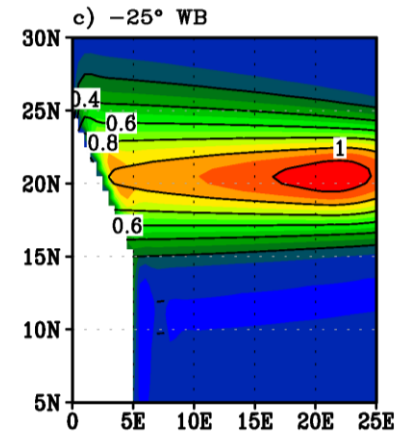
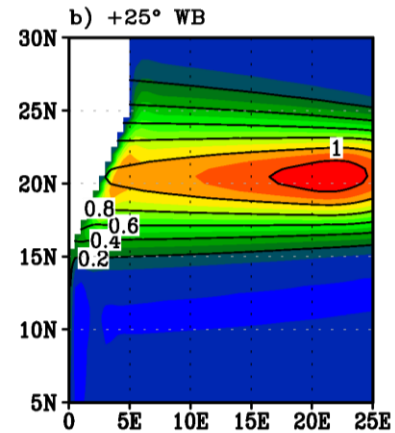
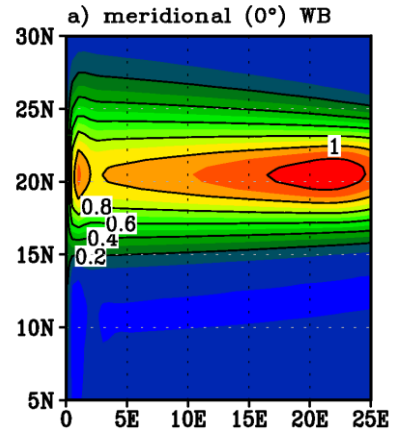


# Steady forcing, slanted western boundary

- Steady forcing center at  $20^{\circ}\text{N}, 30^{\circ}\text{E}$
- Angle between meridian and the western boundary is  $25^{\circ}$  and  $62^{\circ}$ .

Relative Root Mean Square Error [%]

Angle of Western Boundary			
$+25^{\circ}$	$-25^{\circ}$	$+62^{\circ}$	$-62^{\circ}$
2.9%	1.9%	6.7%	5.2%



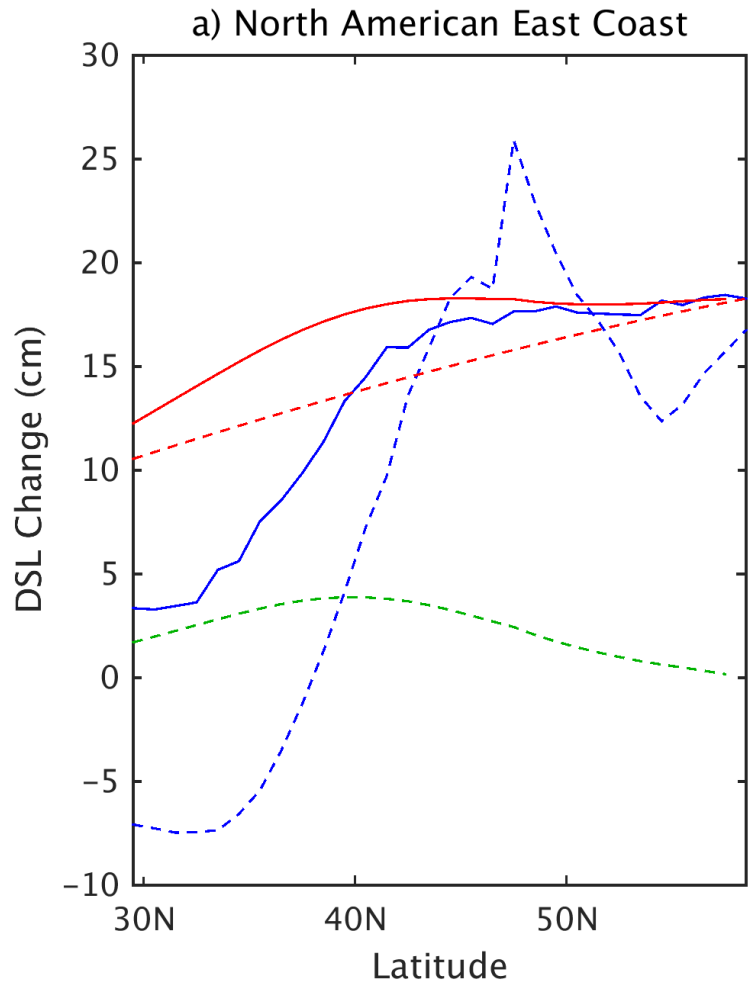
Relative diffusivity

Overall, the theory works very well.

# **Application to sea level change of CMIP5 data**

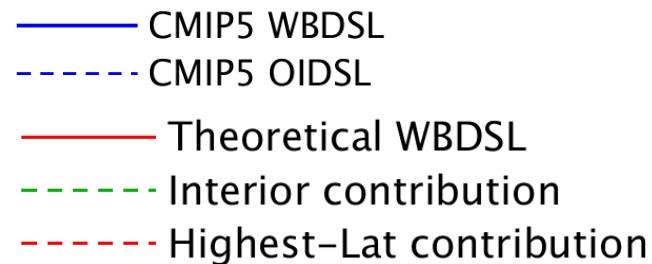
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# DSL Change, CMIP5 MME vs. theory



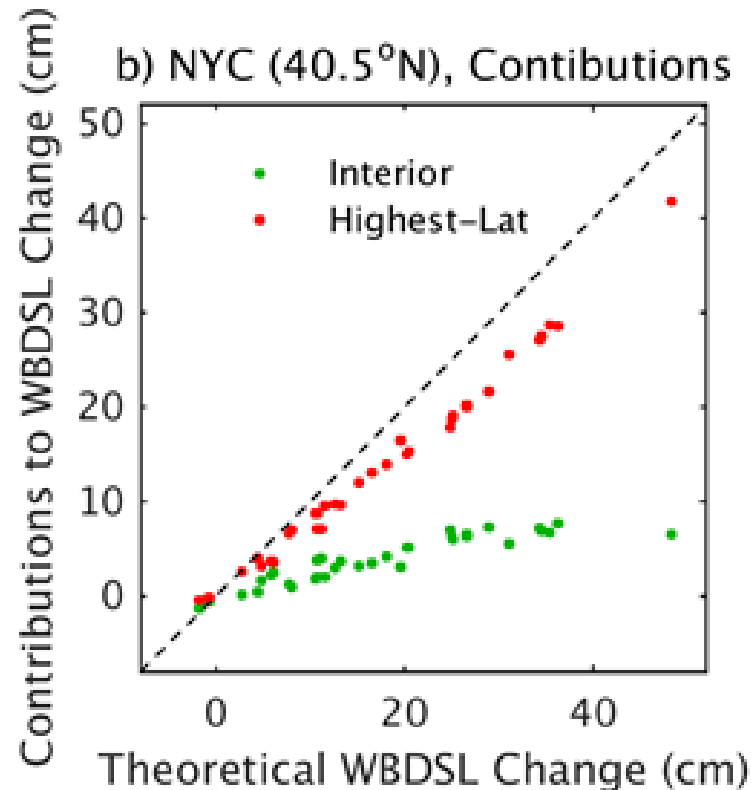
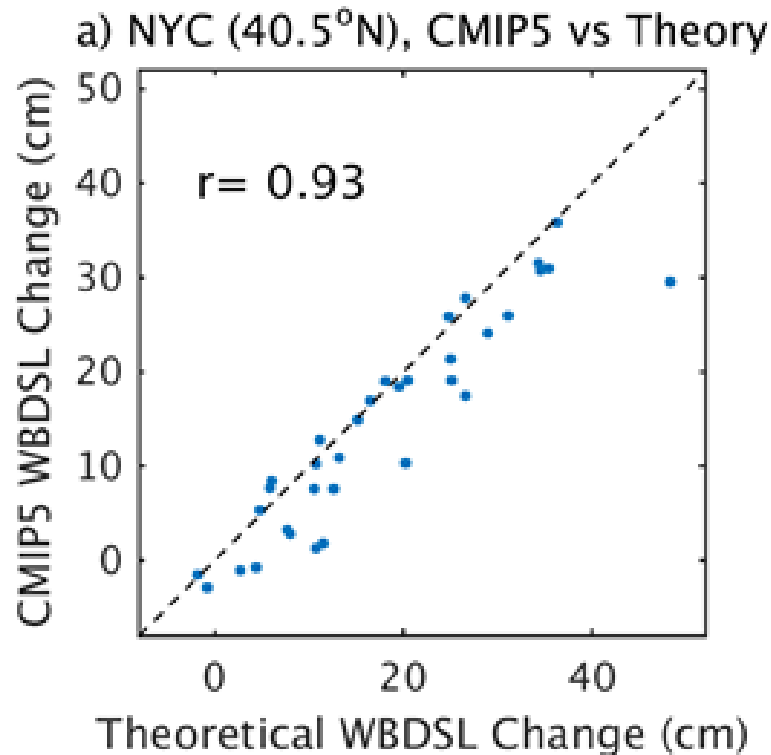
## Method

- 34 models, RCP8.5
- Coastal sea level is pick up at each latitude from each model.
- Ocean interior sea level is 10-deg inward from western boundary.



- Theory reproduces roughly uniform sea-level rise to the north of 40N, and underestimates the reduction toward the south.
- The contribution of the highest latitude WBDSL is dominant.

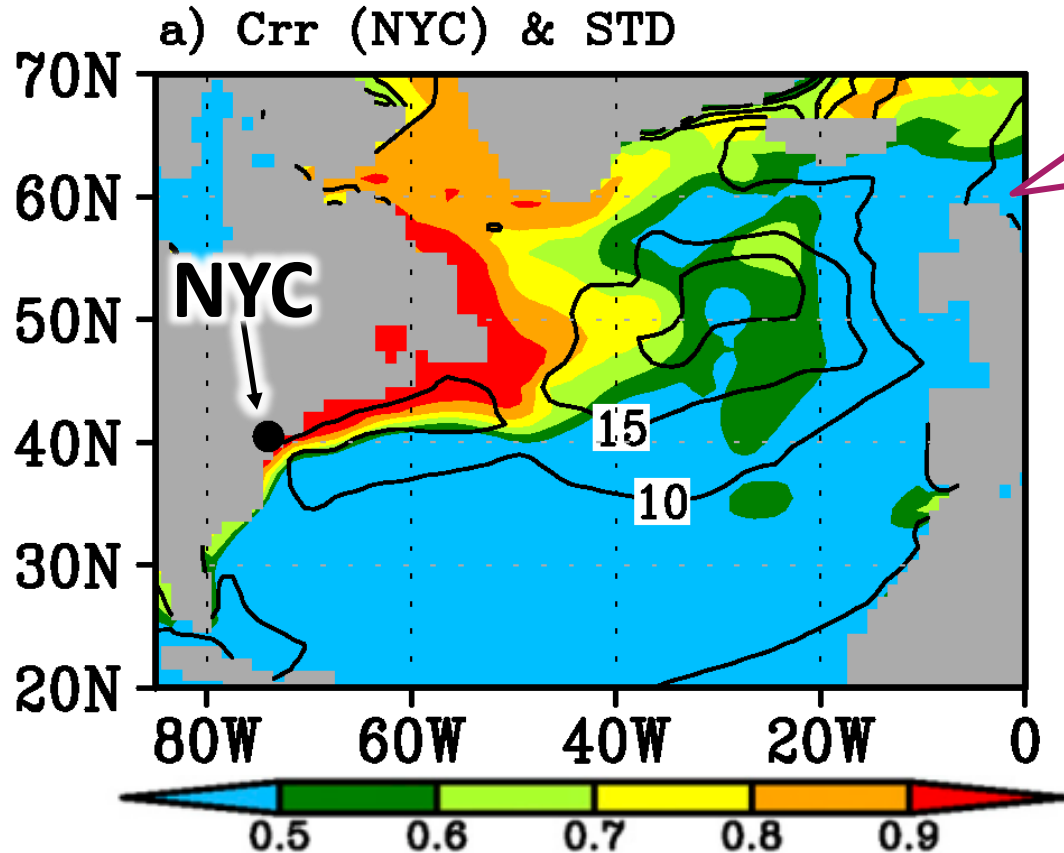
# DSL Change, CMIP5 MME vs. theory



- The theory well reproduces the WBDSSL changes of CMIP5 models at NYC.
- Higher latitude contributions are larger than the interior contribution, and especially dominant for models that have large NYC sea-level rise.

# Inter-model correlation

(an independent analysis from the theory)



Correlations between DSL change in NYC and at each grid point among the 34 models

NYC sea level rise

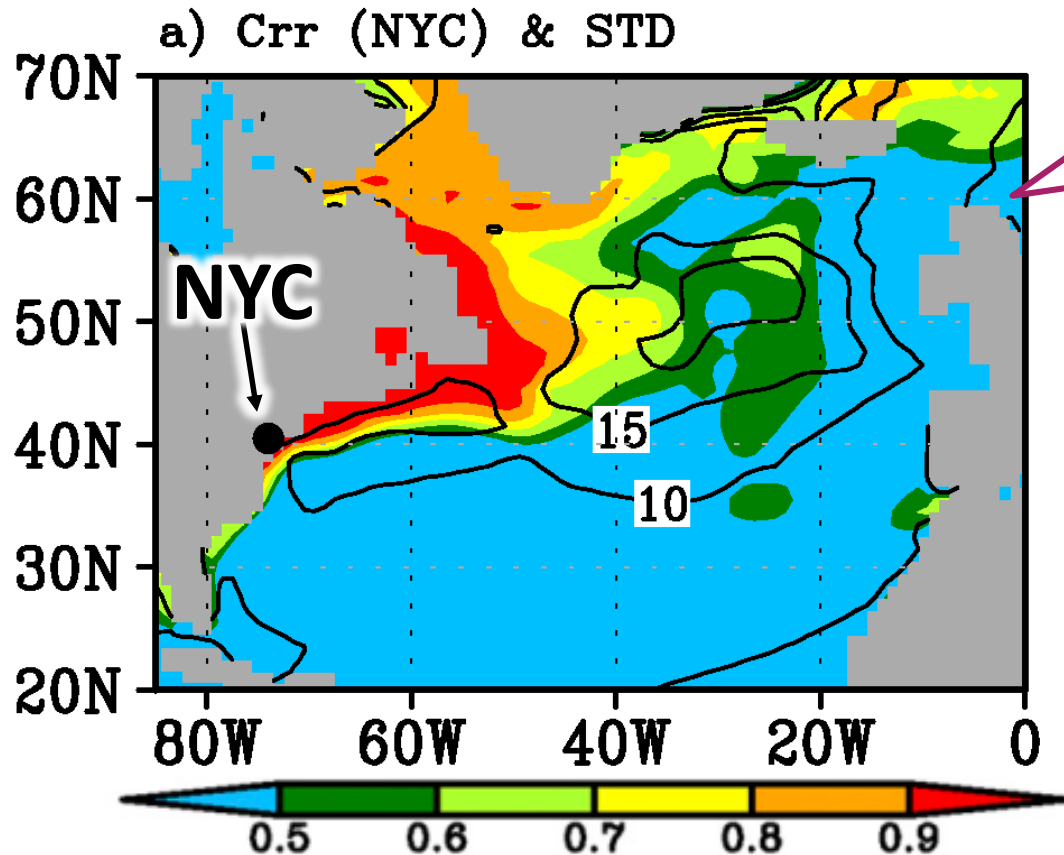
Yin et al. (2009)

- Changes of the Labrador Sea
- Atlantic meridional over-turning circulation (AMOC)



# Inter-model correlation

(an independent analysis from the theory)



Correlations between DSL change in NYC and at each grid point among the 34 models

- Rather than AMOC, the sea level rise in the Labrador Sea has a more direct impact on NYC sea level rise, though the deep convection of the Labrador Sea is an important part of the AMOC.

- We propose a **theory for western boundary sea-level**, which works quite well for a linear reduced gravity model.
- Application of the theory to **CMIP5 data** indicates that the **sea-level rise hot-spot** along the western North Atlantic including NYC is strongly **controlled by sea-level rise in the Labrador Sea**.
- The theory **underestimate the weakened sea-level rise** to the south of NYC, and this should be studied further.