A Stochastic Skeleton Model for the MJO and ENSO

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Motivations

Role of organized/coherent intraseasonal variability and wind bursts on the ENSO? e.g. Madden-Julian Oscillation (MJO) wavetrain during El Niño 2015 onset

(Marshall 2016 ARCCSS)
We model the relationship using a simple **skeleton model atmosphere** for the MJO,

Dry Dynamics: \[ (\partial_t + e)u - yv - \partial_x \theta = 0 \]
\[ yu - \partial_y \theta = 0 \]
\[ (\partial_t + e)\theta - (\partial_x u + \partial_y v) = \overline{Ha} \]

Moisture: \[ (\partial_t + e)q + \overline{Q}(\partial_x u + \partial_y v) = -\overline{Ha} + E_q + \sigma_q \dot{W} \]

Convection: \[ (\partial_t + e)a = \Gamma q(s + a) + \sigma_a \dot{W} \]

where \( a \) is the planetary envelope of convective/synoptic fluctuations.
The skeleton model captures the main features of the MJO: e.g. phase speed \( \approx 5 \, m.s^{-1} \), period \( \approx 40 \) days, intermittent wavetrains and structure.
A Skeleton Model for the MJO

A multiple time approach yields the interannual evolution of the skeleton atmosphere:

\[ u = \bar{u}(x, y, \tau) + u'(x, y, t, \tau) \]

Interannual Atm:

\[-y \bar{v} - \partial_x \bar{\theta} = 0\]
\[ y \bar{u} - \partial_y \bar{\theta} = 0\]
\[-(\partial_x \bar{u} + \partial_y \bar{v}) = \bar{E}_q/(1 - \bar{Q})\]
\[ \bar{H}\bar{a} = s + \bar{E}_q/(1 - \bar{Q})\]

Intraseasonal Atm:

\[(\partial_t + e)u' - yv' - \partial_x \theta' = 0\]
\[ yu' - \partial_y \theta' = 0\]
\[(\partial_t + e)\theta' - (\partial_x u' + \partial_y v') = \bar{H}a'\]
\[(\partial_t + e)q' + \bar{Q}(\partial_x u' + \partial_y v') = -\bar{H}a' + \sigma_q \dot{W}\]
\[(\partial_t + e)a' = \Gamma q'\bar{a} + \sigma_a \dot{W}\]
A skeleton ENSO model is obtained by coupling ocean dynamics and a SST heat budget,

### Interannual Atm:

\[-y \bar{v} - \partial_x \bar{\theta} = 0\]
\[y \bar{u} - \partial_y \bar{\theta} = 0\]
\[-(\partial_x \bar{u} + \partial_y \bar{v}) = E_q/(1 - \bar{Q})\]

\[\bar{H} \bar{a} = s + E_q/(1 - \bar{Q})\]

### Intraseasonal Atm:

\[(\partial_t + e)u' - yv' - \partial_x \theta' = 0\]
\[yu' - \partial_y \theta' = 0\]
\[(\partial_t + e)\theta' - (\partial_x u' + \partial_y v') = \bar{H} a'\]
\[(\partial_t + e)q' + \bar{Q}(\partial_x u' + \partial_y v') = -\bar{H} a' + \sigma_q \dot{W}\]
\[(\partial_t + e)a' = \Gamma q' \bar{a} + \sigma_a \dot{W}\]

### Ocean:

\[\partial_t U - cYV + c\partial_x H = c\gamma(\bar{u} + u')\]
\[YU + \partial_Y H = 0\]
\[\partial_t H + c(\partial_x U + \partial_Y V) = 0\]

### SST:

\[\partial_t T = -\zeta E_q + \eta H\]
\[E_q = \alpha T\]

(solutions are computed for first meridional structures/waves ≈ 350 degrees of freedom)
Model solutions (2000 years) with intermittent El Niño events of varying strength.
Intraseasonal variability increases and expands eastward during El Niño onset.
Lagged regressions on eastern Pacific SST: increased MJO and wind bursts during onset

(Thual et al. 2018 JClim)
Realistic $k - \omega$ power spectra and time-integration of intraseasonal noise by the ocean.

(Thual et al. 2018 JClim)
A Skeleton Model for the ENSO and MJO

Tested for improved stochastic convective parameterizations:
e.g. with nonlinearities, multiplicative noise: \((\partial_t + \lambda)a' = \Gamma q'(\bar{a} + a') + \sqrt{\lambda(\bar{a} + a')}\bar{a}\dot{W}\)
A Stochastic Dynamical Skeleton model for the MJO and ENSO

Role of organized/coherent intraseasonal variability and wind bursts?

Model Formulation:
1) Starting skeleton atmosphere for the MJO
2) Interannual dynamics from a multiple-time approach
3) Shallow-water ocean dynamics and SST heat budget

Features:
1) Intraseasonal variability increases and expands eastward during El Niño onset
2) Realistic $k - \omega$ power spectra and time-integration of intraseasonal noise by the ocean
3) Testbed for improved stochastic convective parameterizations/GCM prototype

Perspectives:
1) Eastern vs Central Pacific El Niño
2) Seasonality of MJO-ENSO interactions
3) Other intraseasonal modes: convectively coupled rossby, kelvin waves, etc.